Graphic-analytical method of construction of the manifolds (surfaces) in the space E_n

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Introduction

The construction and research of the manifolds (surfaces), of the spaces of different dimensions and structural characteristics belong to one of the actual problems of applied geometry. They are widely used in the solution of important applied problems in various areas of science and technology and particularly in the studying of multifactorial processes of multicomponent systems.

Now the sphere of construction surfaces with the main purpose of the studying of manifolds/surfaces (which are the generators of the linear objects or objects of m-order with the main objective - to build a geometric model of the physical surface of the synthesis of new tasks or implementation of new engineering project) is the most interest in science.

Recently the descriptive geometry of multidimensional spaces with access to the parameterization of the theory and the geometry of the enumeration become more important in the methods of surfaces modeling. The using of the complex curved surfaces that satisfy a given engineering - technical requirements is constantly expanding in mechanical engineering. There are different geometrical problems at the design stage of such surfaces: the construction of the surface shape and a constructive receipt of the analytical algorithm for the description of complex cyclic and ruled surfaces, the analysis of their structural characteristics in order to obtain the regression equation, etc.

Method

The methods of enumerative geometry allow determining of the parametric number of curves and surfaces, the dimension of the set conditions, the special characteristics of enumerative elements of objects, entering of the symbolic notation for the terms and conditions of use of symbolic calculus which greatly simplifies the apparatus of the studying.

We propose the following method of constructing of the surfaces in the space En based on the theory and parameterization of enumerative geometry [2]. The method allows carrying out a formal choice of options for types of surfaces in order to identify it with the predetermined conditions. It is presented as a block - scheme (Figure 1).

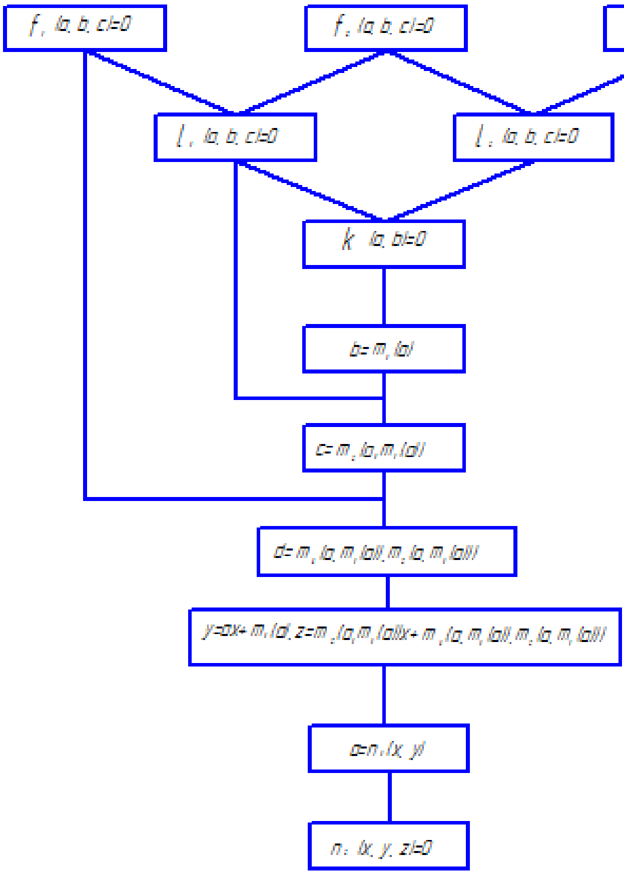

Figure 1. Algorithm for the withdrawal of the mathematical model of a cyclic surface

Structural representation of the derivation of the analytical equation of the circular surface:

1. For example, specify four systems of equations - the conditions for the generator - second-order curve.
2. Express four independent variables on one of the variables from five independent variables.
3. Substitute these expressions in four variables of the system of equations of the surface.
4. Freed from the variable to obtain an analytical representation of a cyclic surface

Algorithm for constructing manifolds in the space E_n

1. Determine the number of parametric, forming a line m-order.

Counting the dimensions of the forming line in a space can be done by the formula (1) the number of parametric [1]::

$$L_{n-1}^{m} = \frac{1}{n!} \cdot \prod_{i=1}^{n} (m+i) - 1, \quad (1)$$

where ***m*** is the order of an algebraic curve,

n is the dimension of the space.

Since n = 3, then (1) is converted to the form (2) [5]:

$$L_2^m = \frac{1}{2!} \cdot (m+1)(m+2) - 1 \qquad (2)$$

since the surface is determined by the one-parameter family of the forming lines of m-order, then the parameter will be equal to the number of:

$$L_2^m - 1 = \frac{(m+1)(m+2)}{2} - 1$$

2. Create a set of geometric conditions of incidence, of the parallelism, of the tangency and determine their dimensions [4]

Assume that the forming line of an analytic curve is represented by a system of equations

$$y = \psi_1(x)$$
$$z = \psi_2(x)$$

Type of conditions	Dimension	Presentation of the conditions	
		symbolic	analytical
The forming line passes through a given point $M(x_1, y_1, z_1)$	2	$e_{3,0}^{1,0}$	$y_1 = \psi_1(x_1)$ $z_1 = \psi_2(x_1)$
The forming line crosses a given line $\begin{cases} y = a_1x + b_1 \\ z = c_1x + d_1 \end{cases}$	1	$e_{3,1}^{1,0}$	$\psi_1(x) = ax + b$ $\psi_2(x) = cx + d$
The forming line of a given line intersects the curve p-order $y = \varphi_1(x)$ $z = \varphi_2(x)$	1	$pe_{3,1}^{1,0}$	$\psi_1(x) = \varphi_1(x)$ $\psi_2(x) = \varphi_2(x)$
The forming line line touch a given plane $z = ax + by + c$	1	$e_{3,1}^{1,0}$	$a = \frac{dy}{dx}$ $b = \frac{dz}{dx}$
The forming line touch a given surface of the p-order $z = F(x, y)$	1	$pe_{3,1}^{1,0}$	$\frac{dy}{dx} = \frac{dF(x,y)}{dx}$ $\frac{dz}{dx} = \frac{dF(x,z)}{dx}$

Table 1. Geometrical conditions

3. Choose the formal features of the number of satisfying assignments to the surface [2] based on the conditions of dimension

4. Check the conditions on the compatibility, determine the order of the constructed surface

5. Submit the conditions in the analytical form

We can consider the formulated algorithm for constructing manifolds (surfaces) in terms of the construction of a cyclic surface, forming the line of the second order.

Results

The forming line of the cyclic surface – is a circle. There are six-parameter set of circles in the space E_3, so, it should be chosen the conditions to construct a cyclic surface. The total dimension of these conditions must be equal to five.

The dimension of the linear objects can be determined by the formula of Grassmann (3):

$$D_n^m = (n - m)(m + 1) \qquad (3)$$

where ***n*** is the dimension of the space in which the Grassmann manifold is regarded,

m is the dimension of the plane (element) which forms the Grassmanns' manifold.

Since the number of parametric plane E_3 is equal to three [6]:

$$D_3^2 = (2+1)(3-2) = 3$$

and the dimension of the circles on the plane is on the basis of the formula (2) for second-order curves and is equal to three, since all the circles in the plane, passing through two fixed points (absolute plane):

$$L_2^m = \frac{1}{2!} \cdot (2 + 1)(2 + 2) - 1 = 5$$

We assume that the plane in which the circle will lie is parallel to the plane parallelism. The dimension of this condition is equal to two.

$$U_{nap} = 1 \cdot 2(3 - 2 - 2 + 1 \cdot 2) = 2$$

The dimension of the intersection of the circle with the rail line is equal to unity $e_{3,2}^{1,0} = 1$

It follows that the total dimension of the geometrical conditions is five.

In Figure 2 one of the forming lines of the cyclic surface is built. Define in the space E_3 two planes of general position: f (a, b) and ABC - and the line c. Since the plane is parallel to the N, horizontal plane of projection, the formal line in the horizontal plane of projection is projected in full size. To construct a circle, using a solution in this particular case, the problem Apolonia is used. The result of the constructions will be a circle, they also form a cyclic surface.

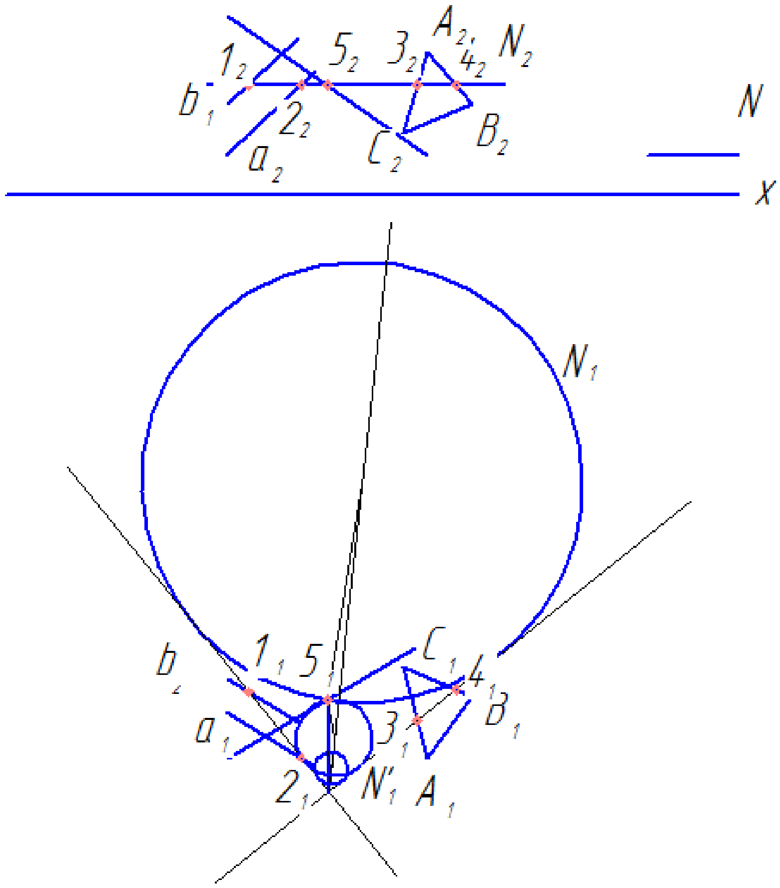

Figure 2. Construction of one of the forming lines of the cyclic surface

Discussion

It follows that the proposed formal algorithm can be used for constructing surfaces in engineering practice. The design of the complex technical curvilinear forms, their calculation and playback require the development of valid mathematical models that implement a particular method of constructing the surface. It raises the questions set in the drawing and studies the properties of the constructed surface which are interconnected by way of their education. Of course, every method of the forming of the surfaces has its advantages and disadvantages which should facilitate the analysis of the choice of a method in each case, the design of technical surfaces. It must be remembered that we need to combine the simplicity of common engineering methods and the possibility of studying their properties inherent to the classical methods of the theory of surfaces in the analysis of the existing methods of constructing surfaces. It allows using the methods of enumerative geometry .It is probably the most acceptable solution to the technical challenges of designing surfaces, that satisfy a prescribed circumstances.

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